

Verification of categorical predictands

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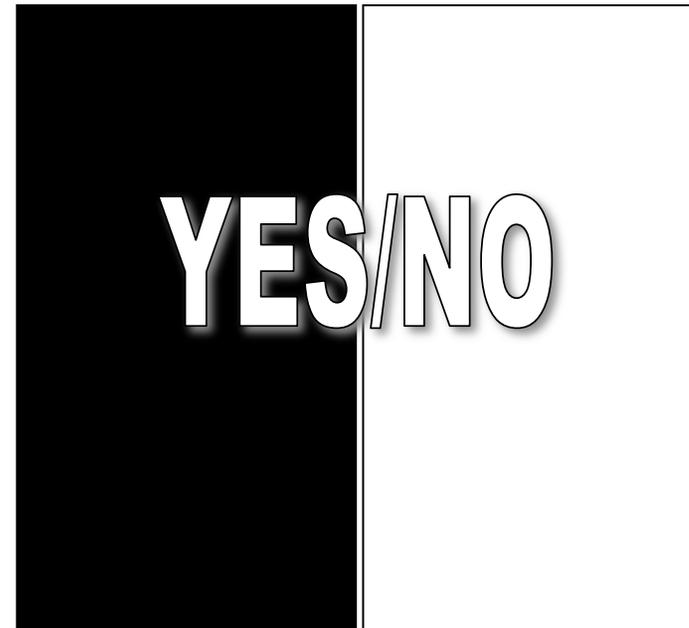
ECMWF



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Outline

- What is a categorical forecast?
- Basic concepts
- Basic scores
- Exercises
- Conclusions



Basic concepts

- Categorical: only one of a set of possible events will occur
- Categorical forecast does not contain expression of uncertainty
- There is typically a one-to-one correspondence between the forecast values and the observed values.
- The simplest possible situation is a 2x2 case or verification of a categorical yes/no forecast: 2 possible forecasts (yes/no) and 2 possible outcomes (event observed/event not observed)



Contingency tables

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	Hit	False alarm	Fc Yes
No	Miss	Correct non-event	Fc No
Marginal total	Obs Yes	Obs No	Sum total

↕

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n



How do we build a contingency table?

Date	24 hour forecast	Observed quantities	
Jan. 1st	0.0	-	→ d
Jan. 2nd	0.4	yes	
Jan. 3rd	1.5	yes	→ a
Jan. 4th	2.0	-	→ b
..			
..			
Mar. 28th	5.0	-	
Mar. 29th	0.0	yes	→ c

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n



Contingency tables

Marginal probability: sum of column or row divided by the total sample size

For example the marginal probability of a yes forecast is:

$$p_x = \Pr(X=1) = 100/2800 = 0.03$$

Tornado forecast	Tornado Observed		
	yes	no	Total fc
yes	30	70	100
no	20	2680	2700
Total obs	50	2750	2800

Sum of rows

Total sample size



Contingency tables

Joint probability: represents the intersection of two events in a cross-tabulation table.

For example the joint probability of a yes forecast and a yes observed:

$$P_{x,y} = \Pr(X=1, Y=1) = 30/2800 = 0.01$$

Tornado forecast	Tornado Observed		
	yes	no	Total fc
yes	30	70	100
no	20	2680	2700
Total obs	50	2750	2800



Basic measures/scores

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n

Frequency Bias Index (Bias)

$$FBI = B = \frac{(a + b)}{(a + c)}$$

- FBI > 1 over forecasting
- FBI < 1 under forecasting

Range: 0 to ∞
Perfect score = 1

Proportion Correct

$$PC = \frac{(a + d)}{n}$$

Range: 0 to 1
Perfect score = 1

- simple and intuitive
- yes and no forecasts are rewarded equally
- can be maximised by forecasting the most likely event all the time



Basic measures/scores

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n

Hit Rate, Probability Of Detection, Prefigurance

$$H = POD = \frac{a}{(a + c)}$$

- sensitive to misses events and hits, only
- can be improved by over forecasting
- complement score Miss Rate $MS=1-H=c/(a+c)$

False Alarm Ratio

$$FAR = \frac{b}{(a + b)}$$

- function of false alarms and hits only
- can be improved by under forecasting

Range: 0 to 1
Perfect score = 1

Range: 0 to 1
Perfect score = 0



Basic measures/scores

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n

Post agreement

$$PAG = \frac{a}{(a + b)}$$

- Complement FAR -> PAG=1-FAR
- not widely used
- sensitive to false alarms and hits

Range: 0 to 1
Perfect score = 1

False Alarm Rate, Probability of False Detection

$$F = \frac{b}{(b + d)}$$

- sensitive to false alarms and correct negative
- can be improved by under forecasting
- generally used with H (POD) to produce ROC score for probability forecasts (see later on in the week)

Range: 0 to 1
Perfect score = 0



Basic measures/scores

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n

Threat Score, Critical Success Index

$$TS = CSI = \frac{a}{(a + b + c)}$$

- takes into account: hits, misses and false alarms
- correct negative forecast not considered
- sensitive to climatological frequency of event

Range: 0 to 1
Perfect score = 1
No skill level = 0

Equitable Threat Score, Gilbert Skill Score (GSS)

$$ETS = \frac{(a - a_r)}{(a + b + c - a_r)}$$

$$a_r = \frac{(a + b)(a + c)}{n}$$

- it is the TS which includes the hits due to the random forecast

Range: -1/3 to 1
Perfect score = 1
No skill level = 0



Basic measures/scores

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n

Hanssen & Kuipper's Skill Score, True Skill Statistic (TSS), Pierce's Skill Score

$$KSS = TSS = H - F = \frac{(ad - bc)}{[(a + c)(b + d)]}$$

- popular combination of H and F
- Measures the ability to separate yes (H) and no (F) cases
- For rare events d is very large -> F small and KSS (TSS) close to POD (H)
- Related to ROC (Relative Operating Characteristic)

Range: -1 to 1
Perfect score = 1
No skill level = 0

Heidke Skill Score

$$HSS = \frac{2(ad - bc)}{[(a + c)(c + d) + (a + b)(b + d)]}$$

- Measures fractional improvements over random chance
- Usually used to score multi-category events

Range: -∞ to 1
Perfect score = 1
No skill level = 0



Basic measures/scores

Event forecast	Event observed		Marginal total
	Yes	No	
Yes	a	b	a + b
No	c	d	c + d
Marginal total	a + c	b + d	a + b + c + d = n

Odds Ratio

$$OR = \frac{ad}{bc}$$

- measures the forecast probability(odds) to score a hit (H) compared to giving a false alarm (F)

$$OR = \frac{\left[\frac{H}{1-H} \right]}{\left[\frac{F}{1-F} \right]}$$

- independent of biases
- unbound

Range: 0 to ∞
 Perfect score = ∞
 No skill level = 1

Odds Ratio Skill Score

$$ORSS = \frac{(ad - bc)}{(ad + bc)} = \frac{OR - 1}{OR + 1}$$

- produces typically very high absolute skill values (because of its definition)
- Not widely used in meteorology

Range: -1 to 1
 Perfect score = 1



Verification history

Tornado forecast	Tornado Observed		Total fc
	yes	no	
yes	30	70	100
no	20	2680	2700
Total obs	50	2750	2800

$$PC = (30 + 2680) / 2800 = 96.8\%$$

$$H = 30 / 50 = 60\%$$

$$FAR = 70 / 100 = 70\%$$

$$B = 100 / 50 = 2$$

Tornado forecast	Tornado Observed		Total fc
	yes	no	
yes	0	0	0
no	50	2750	2800
Total obs	50	2750	2800

$$PC = (2750 + 0) / 2800 = 98.2\%$$

$$H = 0 = 0\%$$

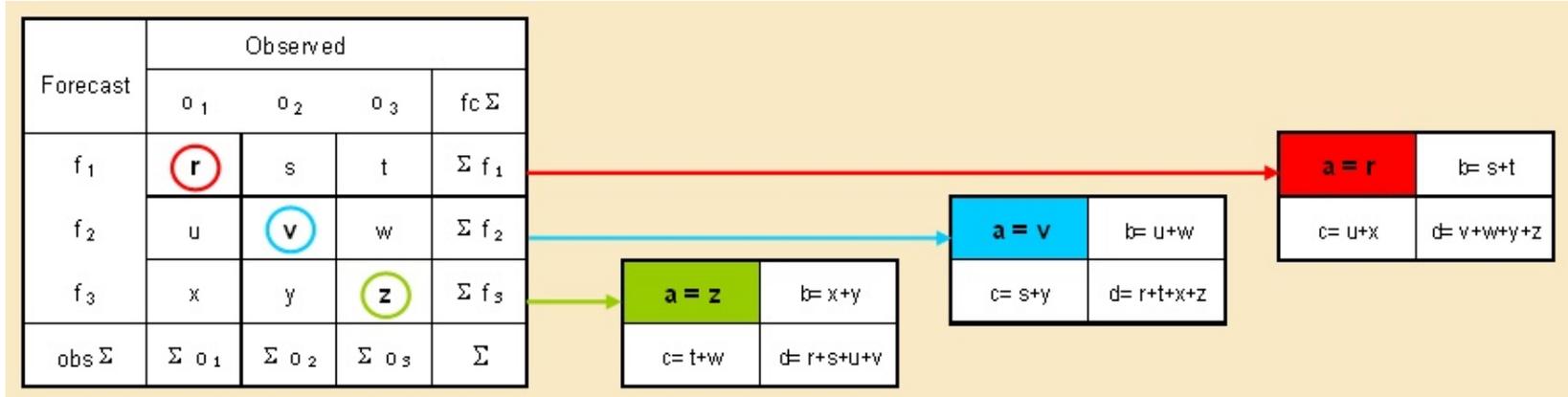
$$FAR = 0 = 0\%$$

$$B = 0 / 50 = 0$$



Multi-category events

- The 2x2 tables can be extended to several mutually exhaustive categories
 - ❖ Rain type: rain/snow/freezing rain
 - ❖ Wind warning: strong gale/gale/no gale
 - ❖ Cloud cover: 1-3 okta/4-7 okta/ >7 okta
- Only PC (Proportion Correct) can be directly generalised
- Other verification measures need to be converted into a series of 2x2 tables



Generalised version of HSS and KSS - measure of improvement over random forecast

$$HSS = \frac{\left\{ \sum p(f_i, o_i) - \sum p(f_i)p(o_i) \right\}}{\left\{ 1 - \sum p(f_i)p(o_i) \right\}}$$

$$KSS = \frac{\left\{ \sum p(f_i, o_i) - \sum p(f_i)p(o_i) \right\}}{\left\{ 1 - \sum (p(\cancel{o}_i))^2 \right\}}$$

f_i



Multi-category events

Clouds forecast	Clouds observed			fc Σ
	0-2	3-5	6-8	
0-2	65	10	21	96
3-5	29	17	48	94
6-8	18	10	128	156
obs Σ	112	37	197	346

~>

No clouds (0-2)	Partly cloudy (3-5)	Cloudy (6-8)
B = 0.86	B = 2.54	B = 0.79
POD = 0.58	POD = 0.46	POD = 0.65
FAR = 0.32	FAR = 0.82	FAR = 0.18
F = 0.13	F = 0.25	F = 0.19
TS = 0.45	TS = 0.15	TS = 0.57

Calculate:

PC = ?

KSS = ?

HSS = ?

results:

PC = 0.61

KSS = 0.41

HSS = 0.37



Summary scores

Gale forecast	Gale observed			Tornado forecast	Tornado observed		
	Yes	No	fc Σ		Yes	No	fc Σ
Yes	15	2	17	Yes	30	70	100
No	11	123	134	No	20	2680	2700
obs Σ	26	125	151	obs Σ	50	2750	2800

Left panel: Contingency table for five months of categorical warnings against gale-force winds (wind speed > 14m/s) **Right panel:** Tornado verification statistics

$B = (a+b)/(a+c)$	<u>0.65</u>	<u>2.00</u>
$PC = (a+d)/n$	<u>0.91</u>	<u>0.97</u>
$POD = a/(a+c)$	<u>0.58</u>	<u>0.60</u>
$FAR = b/(a+b)$	<u>0.12</u>	<u>0.70</u>
$PAG = a/(a+b)$	<u>0.88</u>	<u>0.30</u>
$F = b/(b+d)$	<u>0.02</u>	<u>0.03</u>
$KSS = POD-F$	<u>0.56</u>	<u>0.57</u>
$TS = a/(a+b+c)$	<u>0.54</u>	<u>0.25</u>
$ETS = (a-a)/(a+b+c-a)$	<u>0.48</u>	<u>0.24</u>
$HSS = 2(ad-bc)/[(a+c)(c+d)+(a+b)(b+d)]$	<u>0.65</u>	<u>0.39</u>
$OR = ad/bc$	<u>83.86</u>	<u>57.43</u>
$ORSS = (OR-1)/(OR+1)$	<u>0.98</u>	<u>0.97</u>
	GALE	TORNADO



Example 1

<http://tinyurl.com/verif-training>

Below are two contingency tables representing two completely different sets of forecasts. The left hand table is for 151 Gale forecasts, and the right table is for 2800 forecasts of the occurrence of tornados in the US. Test your understanding of frequency bias using the question below the table.

Gale forecast	Gale observed		
	Yes	No	fc Σ
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		
	Yes	No	fc Σ
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Which of the following statements correctly describes the bias of the two sets of forecasts?

- Gales and tornados are underforecast
- Gales are underforecast and tornados are overforecast
- Gales are overforecast and tornados are underforecast
- Gales and tornados are overforecast



Example 1 -- Answer

Below are two contingency tables representing two completely different sets of forecasts. The left hand table is for 151 Gale forecasts, and the right hand table is for 2800 forecasts of the occurrence of tornados in the US. Test your understanding of frequency bias using the question below the tables:

Gale forecast	Gale observed		fc Σ
	Yes	No	
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		fc Σ
	Yes	No	
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Which of the following statements correctly describes the bias of the two sets of forecasts?

- Gales and tornados are underforecast
- Gales are underforecast and tornados are overforecast
- Gales are overforecast and tornados are underforecast
- Gales and tornados are overforecast

Feedback

Yes, correct: For gales, $B=17/26$ which is less than 1 while for tornados, $B=100/50$ which is greater than 1.



Example 2

is especially true where the event of interest is rare: It may be possible to obtain a higher proportion correct by not forecasting the rare event at all.

Gale forecast	Gale observed		
	Yes	No	fc Σ
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		
	Yes	No	fc Σ
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Question: Determine the PC and the hit rate for the two contingency tables shown above by dragging the correct answer to the appropriate boxes in the table.

	Gale Forecasts	Tornado Forecasts
Hit Rate =		
Proportion Correct =		

0.30

0.40

0.42

0.58

0.60

0.88

0.91

0.97



Example 2 -- answer

is especially true where the event of interest is rare: It may be possible to obtain a higher proportion correct by not forecasting the rare event at all.

Gale forecast	Gale observed		fc Σ
	Yes	No	
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		fc Σ
	Yes	No	
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Question: Determine the PC and the hit rate for the two contingency tables shown above by dragging the correct answer to the appropriate boxes in the table.

correct

	Gale Forecasts	Tornado Forecasts
Hit Rate =	0.58	0.60
Proportion Correct =	0.91	0.97

0.30

0.40

0.42

0.88



Example 3

Thus, F is the fraction of non-events which were forecast as false alarms. The false alarm rate is sometimes called the probability of false detection (POFD). In that sense, it is akin to false positives on a medical test, or on an X-ray. Along with the hit rate, the false alarm rate is used in the relative operating characteristic (ROC) calculations and is related to the Hanssen-Kuipers skill score, described in the next unit. It is not otherwise widely used.

Gale forecast	Gale observed		fc Σ
	Yes	No	
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		fc Σ
	Yes	No	
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Question: Determine the false alarm ratio and false alarm rate for the two contingency tables shown above by dragging the correct answer to the appropriate boxes in the table.

4 / 7

	Gale Forecasts	Tornado Forecasts
False alarm ratio =		
False alarm rate =		

- 0.02
- 0.03
- 0.12
- 0.30
- 0.70
- 0.88



Example 3 -- answer

Thus, F is the fraction of non-events which were forecast as false alarms. The false alarm rate is sometimes called the probability of false detection (POFD). In that sense, it is akin to false positives on a medical test, or on an X-ray. Along with the hit rate, the false alarm rate is used in the relative operating characteristic (ROC) calculations and is related to the Hanssen-Kuipers skill score, described in the next unit. It is not otherwise widely used.

Gale forecast	Gale observed		fc Σ
	Yes	No	
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		fc Σ
	Yes	No	
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Question: Determine the false alarm ratio and false alarm rate for the two contingency tables shown above by dragging the correct answer to the appropriate boxes in the table.

correct		Gale Forecasts	Tornado Forecasts	
False alarm ratio =	0.12	0.70	0.30	
False alarm rate =	0.02	0.03	0.88	



Example 4

Gale forecast	Gale observed		
	Yes	No	fc Σ
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		
	Yes	No	fc Σ
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Determine the CSI, the number of hits by chance (a_r) and the ETS for the two contingency tables shown above by dragging the correct answer to the appropriate boxes in the table.

	Gale Forecasts	Tornado Forecasts
Threat Score		
Hits by chance =		
Equitable Threat Score =		

0.24

0.25

0.43

0.48

0.54

0.60

1.79

2.93

Example 4 -- answer

part of the exercise asks you to calculate the two scores for the two sets of forecasts, then the second part leads you through an interpretation of the results.

Gale forecast	Gale observed		
	Yes	No	fc Σ
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		
	Yes	No	fc Σ
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Determine the CSI, the number of hits by chance (a_p) and the ETS for the two contingency tables shown above by dragging the correct answer to the appropriate boxes in the table.

correct

	Gale Forecasts	Tornado Forecasts
Threat Score	0.54	0.25
Hits by chance =	2.93	1.79
Equitable Threat Score =	0.48	0.24

0.43

0.60



Example 5

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Critical Success Index (CSI) or Threat Score (TS), and Equitable Threat Score (ETS) - Question 2

	Gale Forecasts	Tornado Forecasts
Threat Score	0.54	0.25
Hits by chance =	2.93	1.79
Equitable Threat Score =	0.48	0.24

Using the results obtained above, select the correct answer to the following question

How did the value of the ETS change with respect to the TS?

- For both gales and tornados, the ETS is smaller than the TS
- The ETS for gales is higher, lower for tornados
- The ETS for gales is lower, but higher for tornados
- The ETS is higher for both gales and tornados



Example 5 -- answer

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Critical Success Index (CSI) or Threat Score (TS), and Equitable Threat Score (ETS) - Question 2

	Gale Forecasts	Tornado Forecasts
Threat Score	0.54	0.25
Hits by chance =	2.93	1.79
Equitable Threat Score =	0.48	0.24

Using the results obtained above, select the correct answer to the following question

How did the value of the ETS change with respect to the TS?

- For both gales and tornados, the ETS is smaller than the TS
- The ETS for gales is higher, lower for tornados
- The ETS for gales is lower, but higher for tornados
- The ETS is higher for both gales and tornados

Feedback

Correct. Actually, the ETS must always decrease because the number correct by chance is subtracted from both numerator and denominator



Example 6

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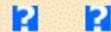
Critical Success Index (CSI) or Threat Score (TS), and Equitable Threat Score (ETS) - Question 3

	Gale Forecasts	Tornado Forecasts
Threat Score	0.54	0.25
Hits by chance =	2.93	1.79
Equitable Threat Score =	0.48	0.24

Looking at the table, are the following statements true or false?

The number correct by chance is greater for gales than for tornados

True False



The decrease in the ETS, when compared to the CSI, is greater for the gales than for the tornados



7 / 7



Example 6 -- answer

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Critical Success Index (CSI) or Threat Score (TS), and Equitable Threat Score (ETS) - Question 3

	Gale Forecasts	Tornado Forecasts
Threat Score	0.54	0.25
Hits by chance =	2.93	1.79
Equitable Threat Score =	0.48	0.24

Looking at the table, are the following statements true or false?

The number correct by chance is greater for gales than for tornados

True False



The decrease in the ETS, when compared to the CSI, is greater for the gales than for the tornados



Feedback

Yes, 2.93 vs. 1.79. Tornados are a rare event, so the chance of guessing the occurrence of a tornado correctly is lower.

Feedback

Correct. The ETS is about .06 lower for the gales and only .01 lower for the tornadoes. Since the TS is typically lower for rare events than for more common events for a particular hit rate (note the hit rates are nearly equal), the adjustment for chance forecasts helps offset this systematic tendency.

7 / 7



Example 7

The KSS is called a skill score, but is not formulated in the usual format. It does express the hit rate relative to the false alarm rate, and will remain positive as long as H is greater than F.

Gale forecast	Gale observed		fc Σ
	Yes	No	
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		fc Σ
	Yes	No	
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Question: Determine the KSS values for the tornado and gale forecasts and fill them in by dragging the correct value to its place in the table below.

	Gale Forecasts	Tornado Forecasts
Hanssen-Kuiper Skill Score =		

0.02

0.03

0.41

0.52

0.56

0.57



Example 7 -- answer

The KSS is called a skill score, but is not formulated in the usual format. It does express the hit rate relative to the false alarm rate, and will remain positive as long as H is greater than F.

Gale forecast	Gale observed		fc Σ
	Yes	No	
Yes	15	2	17
No	11	123	134
obs Σ	26	125	151

Tornado forecast	Tornado observed		fc Σ
	Yes	No	
Yes	30	70	100
No	20	2680	2700
obs Σ	50	2750	2800

Question: Determine the KSS values for the tornado and gale forecasts and fill them in by dragging the correct value to its place in the table below.

correct

	Gale Forecasts	Tornado Forecasts
Hanssen-Kuiper Skill Score =	0.56	0.57

- 0.02
- 0.03
- 0.41
- 0.52



Example 8

The following exercise summarizes all of the scores discussed in this module and is designed to evaluate your ability to interpret the scores. It should be completed after all the units have been completed, except possibly the last unit. The data consists of one year (with 19 cases missing) of categorical rain vs. no rain forecasts at a specific location in Finland

Rain forecast	Rain observed		fc Σ
	Yes	No	
Yes	52	45	97
No	22	227	249
obs Σ	74	272	346



Scores			
B	=	1.31	TS = 0.44
PC	=	0.81	ETS = 0.32
POD	=	0.70	KSS = 0.53
FAR	=	0.46	HSS = 0.48
PAG	=	0.54	
F	=	0.17	

Which of the following statements about the verification scores are true.

	True	False
Rain is a frequent event at this station	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
Rain was overforecast at this station	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
The high frequency of forecasting of rain has led to a high false alarm rate	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
The PC is high (0.81) because forecasting for this dry location is easy	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
The POD is high (0.7) because forecasting for this dry location is easy	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
The forecasts were skilful on average	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>

Example 8 -- answer

Rain forecast	Rain observed		fc Σ
	Yes	No	
Yes	52	45	97
No	22	227	249
obs Σ	74	272	346



Scores			
B	=	1.31	TS = 0.44
PC	=	0.81	ETS = 0.32
POD	=	0.70	KSS = 0.53
FAR	=	0.46	HSS = 0.48
PAG	=	0.54	
F	=	0.17	

Which of the following statements about the verification scores are true.

- | | True | False |
|--|-------------------------------------|-------------------------------------|
| Rain is a frequent event at this station | <input type="checkbox"/> | <input checked="" type="checkbox"/> |
| Rain was overforecast at this station | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| The high frequency of forecasting of rain has led to a high false alarm rate | <input type="checkbox"/> | <input checked="" type="checkbox"/> |
| The PC is high (0.81) because forecasting for this dry location is easy | <input checked="" type="checkbox"/> | <input type="checkbox"/> |
| The POD is high (0.7) because forecasting for this dry location is easy | <input type="checkbox"/> | <input checked="" type="checkbox"/> |
| The forecasts were skilful on average | <input checked="" type="checkbox"/> | <input type="checkbox"/> |

Correct. Rain occurs with a frequency of only about 20% (74/346) at this station

True. The frequency bias is 1.31, greater than 1, meaning over-forecasting.

Correct. It could be seen that the over forecasting is accompanied by high false alarm rate, but the false alarm rate depends

Probably true. The PC gives credits to all Those "easy" correct forecasts of the

The POD is high most likely because the Forecaster has chosen to forecast the

Yes, both the KSS and HSS are well within The positive range. Remember, the standard For the HSS is a chance forecast, which is Easy to beat.



Conclusions

- Definition of categorical forecast
- How to build a contingency table and what are the marginal and joint probabilities
- Basic scores for simple 2x2 tables
- Extension to multi-category events
- Practical exercises



references

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